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1987 J. Phys. A: Math. Gen. 20 1309

(http://iopscience.iop.org/0305-4470/20/5/043)

J. Phys. A: Math. Gen. **20** (1987) 1309–1311. Printed in the UK

ADDENDUM

Hunting for the central charge of the Virasoro algebra: six- and eight-state spin models

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Received 14 October 1986

Abstract. We give here the missing conclusions of our paper with the same title where we gave the critical exponents for the six- and eight-state models with cubic symmetry for several values of the coupling constant. From the finite-size correction to the ground-state energy, we find that the central charges are $c \approx 1.25$ for the six-state model and $c \approx 1.30$ for the eight-state model for all the considered values of the coupling constant. We show that for a special value of the coupling constant the six-state model exhibits $N = 1$ superconformal invariance.

The readers might have noticed that our paper (von Gehlen and Rittenberg 1986a) is lacking the conclusions announced in the introduction. The authors had not noticed that the conclusions were missing in the galley proofs. This is fortunate since in the mean time Blöte *et al* (1986) and Affleck (1986) have given a direct prescription for finding the central c charge of the Virasoro algebra and we can give a new interpretation of our results.

The finite-size corrections to the ground-state energy for periodic boundary conditions ($E_0^{(0)}(0)$) are

$$-\frac{E_0^{(0)}(0)}{N} = A + \frac{\pi}{6}\frac{\xi c}{N^2} + \dots \tag{1}$$

Here N represents the number of sites and A is a non-universal constant. The constant ξ (which fixes the Euclidean time scale) was determined in § 5 for both the six- and eight-state models for three values of the coupling constant ε ($\varepsilon = 0, \frac{1}{2}$ and $\frac{3}{5}$). We have used (1) in order to determine numerically the values of c. The results are shown in table 1.

We notice that in the considered interval of ε the central charge is $c \approx 1.25$ for the six-state model and $c \approx 1.30$ for the eight-state model. The fact that $c \geqslant 1$ explains why the critical exponents vary with the coupling constant ε (see table 7 of von Gehlen

Table 1. Estimates for the central charge c for various values of the coupling ε (the six- and eight-state models).

ε	0	$\frac{1}{3}$	$\frac{3}{5}$
Six-state	1.24 (2)	1.23 (2)	1.21 (2)
Eight-state	1.31 (4)	1.27 (4)	1.31 (4)

0305-4470/87/051311+03$02.50

and Rittenberg (1986a)). A similar situation occurs in the Ashkin–Teller (1943) model (see, for example, Kohmoto *et al* 1981). In the case of the Ashkin–Teller model a very interesting phenomenon occurs, namely for special values of the coupling constant the symmetry is higher than conformal invariance and one finds superconformal invariance (von Gehlen and Rittenberg (1986b)). We are going to look for a similar possibility here. The $N=1$ superconformal algebra allows the following values for the central charge and anomalous dimensions (Friedan *et al* 1985, Bershadsky *et al* 1985, Eichenherr 1985):

$$
\begin{aligned}
c &= \frac{3}{2} - \frac{12}{\tilde{m}(\tilde{m}+2)} \qquad (\tilde{m}=2,3,\ldots) \\
\Delta_{p,q} &= \frac{[p(\tilde{m}+2)-q\tilde{m}]^2-4}{8(\tilde{m}+2)\tilde{m}} + \tfrac{1}{32}[1-(-1)^{p-q}]
\end{aligned}
\tag{2}
$$

where $1 \leq p < \tilde{m}$, $1 \leq q < \tilde{m}+2$, with $p-q$ even in the Neveu–Schwarz sector and odd in the Ramond sector. From (2) we find that for $\tilde{m}=6$ and 7 we get $c=\frac{5}{4}$ and $\frac{55}{42} \approx 1.309$, both compatible with the central charges observed for the six- and eight-state model. From now on we will concentrate on the six-state model ($c=\frac{5}{4}$) where our estimates of the critical exponents are more precise and the number of possible anomalous dimensions is smaller. A close examination of table 7 (von Gehlen and Rittenberg 1986a) shows that, in the vicinity of $\varepsilon=0$, one has a supersymmetric point. In table 2 here we show the predictions for the critical exponents together with the numerical estimates. The agreement between the supersymmetry predictions and the numerical estimates is very good. The only discrepancy is for the energy density operator $\bar{R}^0$ where the finite-size scaling correction terms are always large.

It is interesting to mention that Zamolodchikov and Fateev (1985) have suggested that for a special choice of the coupling constants the six-state Z_6 model should show superconformal invariance with $c=\frac{5}{4}$. Their choice of the coupling constants, however, is not compatible with the cubic symmetry of the models considered here.

Table 2. Theoretical values for the scaling dimensions x corresponding to the finite-size scaling estimates $x_{\rm exp}$ (six-state model with $\varepsilon=0$). The estimates for $\bar{R}^0$ and the $\bar{P}_Q^{\hat{Q}}$ are taken from table 7 (von Gehlen and Rittenberg 1986a). The $\bar{P}_Q^{\hat{Q}}(-1)$ correspond to states having a momentum smaller by one unit than the states $\bar{P}_Q^{\hat{Q}}$. Their estimates are new. The values of $\Delta=\frac{1}{12}, \frac{1}{4}, \frac{7}{12}, \frac{3}{4}, \frac{5}{6}$ correspond to the Neveu–Schwarz sector and the values $\Delta=\frac{5}{96}, \frac{3}{32}, \frac{41}{96}, \frac{23}{32}$ to the Ramond sector for $c=\frac{5}{4}$.

	Δ	$\bar{\Delta}$	s	x	$x_{\rm exp}$
$\bar{R}^0$	$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{2}$	0.62 (6)
$\bar{P}_0^1$	$\frac{5}{96}$	$\frac{5}{96}$	0	$\frac{5}{48} \approx 0.104$	0.102 (5)
$\bar{P}_0^2$	$\frac{1}{12}$	$\frac{1}{12}$	0	$\frac{1}{6} \approx 0.167$	0.19 (1)
$\bar{P}_0^3$	$\frac{3}{32}$	$\frac{3}{32}$	0	$\frac{3}{16} \approx 0.187$	0.21 (3)
$\bar{P}_1^1$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{3}$	0.29 (3)
$\bar{P}_1^1(-1)$	0	$\frac{5}{6}$	$-\frac{5}{6}$	$\frac{5}{6} \approx 0.83$	0.832 (4)
$\bar{P}_1^2$	$\frac{41}{96}$	$\frac{3}{32}$	$\frac{1}{3}$	$\frac{25}{48} \approx 0.52$	0.5 (1)
$\bar{P}_1^2(-1)$	$\frac{5}{96}$	$\frac{23}{32}$	$-\frac{2}{3}$	$\frac{37}{48} \approx 0.771$	0.753 (2)
$\bar{P}_1^3$	$\frac{7}{12}$	$\frac{1}{12}$	$\frac{1}{2}$	$\frac{2}{3}$	0.63 (6)
$\bar{P}_2^2$	$\frac{3}{4}$	$\frac{1}{12}$	$\frac{2}{3}$	$\frac{5}{6}$	0.84 (6)
$\bar{P}_2^2(-1)$	$\frac{1}{4}$	$\frac{7}{12}$	$-\frac{1}{3}$	$\frac{5}{6}$	0.84 (3)

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